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ON STRESS DISCONTINUITIES AND EXTREMUM THEOREMS FOR A COMPRESSIBLE PLASTIC SOLID

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The stress discontinuities are investigated for the model of a solid described in [1, 2]. A maximum principle is formulated for the energy dissipation and it is shown that the strain rates vanish under convex flow conditions on the surfaces of stress discontinuity. Relationships connecting the stress tensor components on both sides of the surface of discontinuity are derived.

The discontinuities of the state of stress in a three-dimensional ideal rigidly plastic material have been investigated in [3]. Relationships on the surfaces of discontinuitiy in the state of stress, imposing constraints on the stress tensor discontinuity, are derived for an arbitrary plasticity condition [3], and corollaries of these relationships are obtained for the Mises and Tresca plasticity conditions. Stress discontinuities in the case of the dependence of the plasticity condition on the first invariant of the stress tensor have been examined in [4].

Extremum principles in the theory of an incompressible plastic solid have been examined by a number of authors (see [5]). The limit load properties for structural systems, based on extremum principles, were first exposed in [5, 6].

Kinematic and static theorems concerning the limit load for a compressible rigidly plastic solid are proved below on the basis of the formulated maximum principle for the energy dissipation rates.

1. Let us consider an isotropic rigidly-plastic material whose plasticity condition is given as

$$\Phi(\sigma, \Sigma_2, \Sigma_3) = 0, \quad \sigma = {}^1/_3\sigma_{ii} \tag{1.1}$$

Here σ is the first invariant of the stress tensor, Σ_2, Σ_3 are, respectively, the second and third invariants of the stress deviator.

The associated flow law for a rigidly plastic solid is (ε_{ij}' are the strain rate tensor components)

$$\varepsilon_{ij}' = \lambda_1 p_{ij}, \quad \lambda_1 \geqslant 0, \quad p_{ij} = \partial\Phi / \partial\sigma_{ij}$$

Let us assume that the stress function and its associated flow law [7] have moreover been defined for the material under consideration

$$f = \varphi(\sigma) - e = 0, \quad e = {}^1/_3 e_{ii} \tag{1.2}$$

$$\varepsilon_{ij}'' = \lambda_2 q_{ij}, \quad q_{ij} = \partial f / \partial\sigma_{ij} \tag{1.3}$$

Here e_{ij} is the plastic strain tensor, $\varphi(\sigma)$ is some empirical dependence, ε_{ij}'' are the strain rate tensor components, and λ_2 is an undetermined multiplier.

It has been noted in [2, 7] that the loading function (1.2) is interpreted in the stress space by a plane of constant hydrostatic pressure whose position is determined by the magnitude of the volume strain e.

The loading plane (1.2) in combination with the flow surface (1.1) froms a piecewise-smooth loading surface, at whose angular points the associated plastic flow law [1, 2]

$$\varepsilon_{ij} = {}^1/_2\, (v_{i,j} + v_{j,i}) = \lambda_1 p_{ij} + \lambda_2 q_{ij} \tag{1.4}$$

is satisfied.

Differentiating the loading function (1.2) with respect to time and taking into account the flow law (1.3), we obtain the expression

$$\lambda_2 = 3d\sigma \,/\, dt \tag{1.5}$$

for the undetermined multiplier λ_2.

2. Let us assume that some surface S exists in a deformable solid with the associated flow law (1.4), on which the stress and strain rate tensor components generally undergo a discontinuity while the displacement velocities are continuous. Let us also consider the material to be in the plastic state on both sides of the surface S.

From the continuity condition for the contact stresses on the surface S there follows

$$[\sigma_{ij}]\, \nu_j = (\sigma_{ij}^+ - \sigma_{ij}^-)\, \nu_j = 0 \tag{2.1}$$

Here the plus and minus superscripts denote the stresses on opposite sides of the surface S, and ν_i is the unit vector normal to the surface S.

The geometric compatibility conditions on the surface S are [8]

$$[\varepsilon_{ij}] = {}^1/_2\, (\omega_i \nu_j + \omega_j \nu_i) = [\lambda_1 p_{ij} + \lambda_2 q_{ij}], \quad \omega_i = [v_{i,j}]\, \nu_j \tag{2.2}$$

Let us define a local coordinate system x_i at a point on the surface S such that the normal ν_i would coincide with the direction of the x_3-axis. Then

$$\nu_1 = \nu_2 = 0, \qquad \nu_3 = 1 \tag{2.3}$$

In the local coordinate system (2.3) there follows from (2.1) and (2.2)

$$[\sigma_{i3}] = 0, \quad [\varepsilon_{11}] = [\varepsilon_{12}] = [\varepsilon_{22}] = 0 \tag{2.4}$$

We obtain on the surface S from (2.4)

$$[\sigma_{ij}][\varepsilon_{ij}] = 0 \tag{2.5}$$

It can be established from the form of the physical relationships (1.4) that the strain rate vector is not orthogonal to the flow surface for the body model under consideration. This circumstance is caused by the capacity of the material to change its volume irreversibly independently whether the state of stress satisfies the plasticity condition (1.1) or not.

In contrast to the deviator

$$\varepsilon_{ij}^\circ = \varepsilon_{ij} - {}^1/_3\, (\lambda_1 p_{kk} + \lambda_2 q_{kk})\, \delta_{ij}$$

let us agree to call the quantity $\varepsilon_{ij}^* = \varepsilon_{ij} - {}^1/_3 \lambda_2 q_{kk} \delta_{ij}$ the incomplete strain rate deviator.

In the case of the model under consideration, the Drucker postulate should be formulated with respect to the incomplete strain rate deviator components

$$\varepsilon_{ij}^* = \varepsilon_{ij} - \varepsilon\delta_{ij}, \qquad \varepsilon = \frac{d\varphi}{d\sigma}\frac{d\sigma}{dt}$$

since the vector of the incomplete strain rate deviator is proportional to the vector-gradient to the flow surface (1.1). As will be shown below, this circumstance affords the possibility of formulating a maximum principle for the energy dissipation rate for plastic distortion accompanied by "associated" compressibility and irreversible compression independently.

There follows for non-concave flow surfaces from the maximum principle for the energy dissipation rate, which is a corollary of the Drucker postulate formulated for the component ε_{ij}^*

$$(\sigma_{ij} - \sigma_{ij}^\circ)\, \varepsilon_{ij}^* \geqslant 0 \qquad (2.6)$$

Here σ_{ij} are the real values of the stress components corresponding to a given distribution of the quantities ε_{ij}^* and σ_{ij}° are the components of any possible state of stress satisfying the inequality

$$\Phi(\sigma^\circ,\ \Sigma_2^\circ,\ \Sigma_3^\circ) \leqslant 0$$

In the case of strictly convex flow surfaces, the inequality (2.6) can be strengthened and written as

$$(\sigma_{ij} - \sigma_{ij}^\circ)\, \varepsilon_{ij}^* > 0 \qquad (2.7)$$

Let us note that relationships (2.6) and (2.7) express only the maximum principle for the rate of energy dissipation, expended in the plastic distortion and the associated volume change of the body.

An additional maximum principle for the dissipation rate relative to the volume flow described by (1.3) should be formulated for materials capable of altering their volume under hydrostatic compression. We formulate the maximum principle of the energy dissipation rate under hydrostatic compression as follows. The inequality

$$(\sigma - \sigma^\circ)\, \varepsilon \geqslant 0 \qquad (2.8)$$

holds for a fixed value of the volume strain e for any given value of the rate of volume change ε.

Here σ is the real value of the hydrostatic pressure corresponding to a given value of ε, and σ is any possible value of the hydrostatic pressure satisfying the inequality

$$f(\sigma^\circ,\ e) \leqslant 0 \qquad (2.9)$$

The inequality (2.8) imposes a constraint on the form of the volume loading function (1.2).

Indeed, $\varphi(\sigma^\circ) \leqslant \varphi(\sigma)$ follows from (2.9). Since the function $\varphi(\sigma)$ is monotonic (from a qualitative picture of the compression), then $\sigma - \sigma^\circ > 0$, and $(d\sigma / dt > 0)$ results from (2.8) for a loading $d\varphi / d\sigma \geqslant 0$, i.e. there can be no sections shown by the dashes (see Fig. 1) on the curve $e = \varphi(\sigma)$.

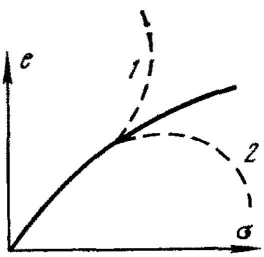

Fig. 1

The dashed section *1* means that an irreversible volume strain originates in the body under the mean stress being removed, i.e. any additional mean stress $\delta\sigma$ does negative work $\delta W = \delta\sigma\delta e$ on the strain increment δe. By analogy with theories of plasticity of an incompressible body, we shall call a compressible material with the property $\delta W < 0$ unstable. A plastic body with the volume loading functions shown by the solid lines in Fig. 1 is an example of a stable compressible material for which $\delta W > 0$. The dashed section *2* evidently contradicts the law of conservation of energy.

Therefore, the additional maximum principle of the energy dissipation rate, written as (2.8), imposes a stability condition on the properties of a compressible material.

If the curve $e = \varphi(\sigma)$ contains no rectilinear sections on which the value of ε can correspond to various points of the volume loading curve according to the relationship

$$\varepsilon = \frac{d\varphi}{d\sigma}\frac{d\sigma}{dt}$$

then the inequality (2.8) may be given the form

$$(\sigma - \sigma^\circ)\, \varepsilon > 0 \tag{2.10}$$

Using the maximum principle for the energy dissipation rate in the form (2.7), on the surface S, we can write

$$[\sigma_{ij}][\varepsilon_{ij}] - 3\,[\sigma][\varepsilon] > 0 \tag{2.11}$$

On the surface S we have from (2.5) and (2.11)

$$(\sigma^+ - \sigma^-)\, \varepsilon^+ + (\sigma^- - \sigma^+)\, \varepsilon^- < 0 \tag{2.12}$$

On the other hand, there follows from the maximum principle (2.8) of the energy dissipation rate

$$(\sigma^+ - \sigma^-)\, \varepsilon^+ \geqslant 0, \qquad (\sigma^- - \sigma^+)\, \varepsilon^- \geqslant 0 \tag{2.13}$$

Comparing (2.12) and (2.13), we conclude that $\varepsilon^+ = \varepsilon^- = 0$ on the surface S. From this and from the relations (2.5) and (2.11) we find

$$\varepsilon_{ij}^+ = \varepsilon_{ij}^- = 0 \tag{2.14}$$

Therefore, for convex flow surfaces the strain rate tensor components vanish on the stress surface of discontinuity within the framework of the body model under consideration.

From the associated flow law (1.4) and (2.14) there follows

$$\lambda_1^\pm = \lambda_2^\pm = 0 \tag{2.15}$$

Multiplying the relationship (2.2) by ν_j and adding over repeated subscripts and taking account of (2.15), we obtain $\omega_i = 0$. It hence follows that the first derivatives of the displacement velocities are continuous on the surface S.

3. As proposed in [3], the equation of the associated flow law (1.4) should be differentiated with respect to the coordinates $x_k, x_m, \ldots, x_l$ and the Hadamard geometric higher order compatibility conditions should be used to obtain constraints on the stress discontinuities.

Performing these operations, we obtain on the surface S

$$[c_{ij}] = {}^1\!/_2\,(a_i\nu_j + a_j\nu_i) = [\varkappa p_{ij} + \theta q_{ij}] \tag{3.1}$$

$$c_{ij} = \varepsilon_{ij,\,k\ldots l}\nu_k \ldots \nu_l, \quad a_i = [v_{i,\,jk\ldots l}]\,\nu_j\nu_k \ldots \nu_l$$

$$\varkappa = \lambda_{1,\,k\ldots l}\nu_k \ldots \nu_l, \quad \theta = \lambda_{2,\,k\ldots l}\nu_k \ldots \nu_l$$

By multiplying (3.1) by ν_j and subsequently adding over repeated subscripts, we determine the quantities

$$a_i = 2\,[\varkappa p_{ik} + \theta q_{ik}]\,\nu_k - [\varkappa p_{kk} + \theta q_{kk}]\,\nu_i \tag{3.2}$$

Taking account of (3.2), the relationships (3.1) become

$$[\varkappa p_{ik} + \theta q_{ik}]\,\nu_k\nu_j + [\varkappa p_{jk} + \theta q_{jk}]\,\nu_k\nu_i - [\varkappa p_{kk} + \theta q_{kk}]\,\nu_i\nu_j = [\varkappa p_{ij} + \theta q_{ij}] \tag{3.3}$$

Only three among the six relationships (3.3) are linearly independent since the system (3.3) is converted into a single equation after being multiplied by $\nu_i\nu_j$.

The material on both sides of the surface S is in the limit state, hence we can write

$$[\Phi(\sigma, \Sigma_2, \Sigma_3)] = \Phi(\sigma^+, \Sigma_2^+, \Sigma_3^+) - \Phi(\sigma^-, \Sigma_2^-, \Sigma_3^-) = 0 \qquad (3.4)$$
$$[f(\sigma, e)] = [\varphi(\sigma)] - [e] = 0$$

For a compressible medium the continuity condition of the medium is

$$d\rho / dt + \rho v_{i,i} = 0, \quad \rho = \rho(\sigma), \quad v_{i,i} = \lambda_1\Phi_{,\sigma} + \lambda_2 f_{,\sigma}$$

where $\rho(\sigma)$ is the density of the medium as a function of the pressure.

Taking (1.5) into account, we obtain from the continuity equation

$$-\gamma(\sigma)\lambda_2 + v_{i,i} = 0, \quad \gamma(\sigma) = -\frac{1}{3\rho}\frac{d\rho}{d\sigma}$$

Differentiating this relationship with respect to the coordinate $x_k x_m, \ldots, x_l$ the same number of times as in deriving (3.3), and using (3.1) we find on the surface S

$$[\theta\{\gamma(\sigma) - q_{33}\}] = [\varkappa p_{kk}] \qquad (3.5)$$

Therefore, we have a closed system of nine equations (2.1), (3.1), (3.4) and (3.5) for the determination of σ_{ij}^-, $\varkappa^-$, θ^-, e^-. In the canonical coordinate system (2.3), this system of equations becomes

$$[\Phi(\sigma, \Sigma_2, \Sigma_3)] = 0, \quad [f(\sigma, e)] = 0$$
$$[\sigma_{i3}] = 0, \quad [\varkappa p_{11} + \theta q_{11}] = [\varkappa p_{12}] = [\varkappa p_{22} + \theta q_{22}] = 0$$
$$[\theta\{\gamma(\sigma) - q_{33}\}] = [\varkappa p_{33}]$$

Let us note that the discussion presented is valid also for plastic bodies whose flow conditions are convex and independent of the hydrostatic part of the stress. In this case we set $\varkappa p_{kk} = 0$ in (3.3).

4. We consider a plastic medium whose limit state is described by the function

$$\psi(S_{ij}) = k^2, \quad S_{ij} = \sigma_{ij} - \sigma\delta_{ij}, \quad k = \text{const} \qquad (4.1)$$

and the volume flow by (1.3).

Let the medium of volume ω under consideration be bounded by the surface $\Sigma = \Sigma_f + \Sigma_v$, where external loads $p_i = p_i^\circ$ are given on the part Σ_f of the body surface while the displacements velocities $v_i = v_i^\circ$ are given on the remaining part Σ_v. For simplicity, let us still assume that the velocity fields v_i are continuous in the whole body volume. Then the equation of the rates of virtual work for real stress and velocity fields is written as

$$\int_\omega (S_{ij}\varepsilon_{ij}^* + 3\sigma\varepsilon)\, d\omega - \int_\Sigma p_i v_i \, d\Sigma = 0 \qquad (4.2)$$

in the case of quasistatic flow of a compressible medium.

Let us consider an arbitrary, kinematically admissible, velocity field v_i' satisfying the continuity condition

$$d\rho/dt + \rho v_{i,i}' = 0, \quad \rho = \bar{\varkappa}(\sigma) \qquad (4.3)$$

and kinematic constraints on the part of the surface Σ_v.

According to the Cauchy equations

$$\varepsilon_{ij} = {}^1/_2\,(v'_{i,\,j} + v'_{j,\,i})$$

components of the strain rate deviator $\varepsilon_{ij}^{*\prime}$ and the volume change rate ε' will correspond to the velocity field v_i'. According to the associated flow law, the stress deviator S_{ij}^* which does not generally satisfy the equilibrium equations will correspond the the components of $\varepsilon_{ij}^{*\prime}$.

The kinematically admissible magnitude of the volume change rate ε' determines the relative change in volume of the medium within the time t [9]

$$e' = \int_0^t \varepsilon'\, d\tau$$

This last integral is computed along the motion trajectories of the material particles. In the case of homogeneous simple strain, the quantity e' can be determined as the sum of the principal logarithmic strains [9]

$$e' = \ln \frac{X_1}{X_1^\circ} + \ln \frac{X_2}{X_2^\circ} + \ln \frac{X_3}{X_3^\circ} \tag{4.4}$$

Here X_i° $(i = 1, 2, 3)$ are the initial lengths of the segments X_i which are the Lagrange coordinates determined for known v_i' (x_i, t) from the relations

$$dX_i \,/\, dt = v_i' \tag{4.5}$$

By integrating (4.3) we obtain the relation [9]

$$\rho = \rho_0 \exp\,(-\,e'),\quad \rho_0 = \rho\,|_{\,t=0} \tag{4.6}$$

Determining the quantities X_i from (4.5) and substituting the value of the volume strain e' computed by means of (4.4) into the continuity equation (4.6), we find the magnitude of the hydrostatic pressure σ^* corresponding to the velocity field v_i'

$$\sigma^* = \bar{\varkappa}^{-1}\,[\rho_0 \exp\,(-\,e')]$$

Here $\bar{\varkappa}^{-1}$ is the inverse function relative to $\bar{\varkappa}$.

The hydrostatic pressure σ^* thus determined does not generally satisfy the equilibrium equations in the general strain case.

Let us note that a method of determining the hydrostatic pressure matched with a discontinuous kinematically-admissible velocity field was examined in [10] for the plane strain case.

We consider the case of loading a body when the loads p_i° on the part of the surface Σ_f grow in proportion to a parameter n, i.e. $p_i^\circ = nq_i^\circ$ (q_i° is some fixed load distribution on Σ_f). Let n_0 denote the value of the parameter n for which the limit state of the body is achieved. Moreover, let us assume that $v_i^\circ = 0$ on the part of the body surface Σ_v.

The surface loads $p_i^* = n_k\, q_i^\circ$ (n_k is the kinematic component) correspond to the stresses $\sigma_{ij}^* = S_{ij}^* + \sigma^*\delta_{ij}$ corresponding to the kinematically admissible velocities v_i'. For the kinematically admissible velocity field v_i', Eq. (4.2) is also valid, therefore

$$n_0 I = \int_\omega (S_{ij}\varepsilon_{ij}^{*\prime} + 3\sigma\varepsilon')\, d\omega, \qquad I = \int_{\Sigma_f} q_i^\circ v_i'\, d\Sigma \tag{4.7}$$

where S_{ij}, σ are components of real stress state imparting the limit state of the body. Equation (4.2) is also satisfied in the case of the fields S_{ij}^*, σ^*, $\varepsilon_{ij}^{*\prime}$, ε', v_i' and has the form

$$n_k I = \int_\omega (S_{ij}^* \varepsilon_{ij}^{*\prime} + 3\sigma^* \varepsilon') \, d\omega \tag{4.8}$$

Subtracting (4.7) from (4.8), we find

$$(n_k - n_0) I = \int_\omega [(S_{ij}^* - S_{ij}) \varepsilon_{ij}^{*\prime} + 3(\sigma^* - \sigma) \varepsilon'] \, d\omega \tag{4.9}$$

For the plasticity condition (4.1), the inequality (2.6) is written as

$$(S_{ij} - S_{ij}^\circ) \varepsilon_{ij}^* \geqslant 0, \quad \varepsilon_{ij}^* = \varepsilon_{ij} - \varepsilon\delta_{ij} \tag{4.10}$$

In conformity with the inequalities (2.8) and (4.10) the right side of (4.9) is non-negative. Then taking account of the positivity of the power of the given loads on Σ_f, we find $n_0 \leqslant n_k$, i.e. for a compressible plastic material with arbitrary convex flow surface, the coefficient of the ultimate load n_0 cannot be greater than the kinetic coefficient n_k.

Let us note that in the case of the presence of discontinuous velocity fields the additional part of the power dissipation on the surfaces of velocity vector discontinuity should be taken into account in (4.2). The power dissipation on surfaces of velocity discontinuity was determined in [11] for a compressible Mises material. The proof of the kinematic theorem in this case is no different, in principle, from that presented above and the quantity n_k has the form

$$n_k = \frac{1}{I} \left[\int_\omega (kH' + 3\sigma^* \varepsilon') \, d\omega + \sum_i \int_{S_i} (\gamma' + \sigma^* [v_z']) \, dS \right], \quad H' = (2\varepsilon_{ij}^{*\prime} \varepsilon_{ij}^{*\prime})^{1/2}$$

$$\gamma' = k / \sqrt{3} \{3 (v_x'^{+} - v_x'^{-})^2 + 3 (v_y'^{+} - v_y'^{-})^2 + 4 (v_z'^{+} - v_z'^{-})^2\}^{1/2}, \quad [v_z'] = v_z'^{+} - v_z'^{-}$$

Here S_i $(i = 1, 2, \ldots)$ is the surface of velocity discontinuity, (x, y, z) is a local coordinate system on the surface S_k, where the z-axis is directed along the normal to S_k, and $v_i'^{\pm}$ are values of the velocity on different sides of the surface of discontinuity.

We examine a statically possible stress field σ_{ij}', which satisfies the relationships

$$\sigma_{ij, j}' = 0, \quad \psi(S_{ij}') \leqslant K^2, \quad e - \varphi(\sigma') \geqslant 0$$

and the boundary conditions on $\Sigma_f : p_i' = n_s q_i^\circ$ (n_s is the static coefficient). For the actual velocity distribution and the statically possible field of stress we have from (4.2)

$$n_s I = \int_\omega (S_{ij}' \varepsilon_{ij}^* + 3\sigma' \varepsilon) \, d\omega \tag{4.11}$$

Using the inequalities (2.8) and (4.10), we find $n_s \leqslant n_0$ from (4.2) and (4.11), i.e. the ultimate load coeffcient n_0 cannot be less than the static coefficient n_s for a rigidly plastic medium irreversibly compressible because of the effect of hydrostatic pressure.

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SYSTEM OF ARBITRARILY ORIENTED LONGITUDINAL SHEAR CRACKS IN AN ELASTIC SOLID

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The following problems on determining the stresses around rectilinear longitudinal shear cracks are examined by the method of singular integral equations: a system of arbitrarily arranged cracks in an unbounded or semi-bounded solid, a periodic system of cracks of arbitrary orientation in infinite and semi-infinite spaces.

The simply-connected domain is usually considered in the investigations [1–9] devoted to a study of the stress distribution around longitudinal shear cracks, when the solution of the problem can be obtained by conformal mapping. If the domain occupied by the solid is multiconnected, then the existing solutions are limited to comparatively simple cases of collinear [1 – 3] or parallel [2–5, 8, 9] cracks.

The problem of determining the stresses in an infinite solid containing arbitrarily arranged rectilinear longitudinal shear cracks is reduced below to a system of integral equations in the general case. This permits the solution of a number of new problems of mathematical theory of cracks. The appropriate problems of